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Crosswind Response of Tall Buildings with Nonlinear Aerodynamic Damping under Nonstationary Wind Excitations

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Abstract

This study presents an analysis framework for crosswind response of tall buildings under nonstationary wind excitations. The aerodynamic forces are modelled in terms of force parameters under stationary wind excitation with consideration of time-varying mean wind speed in a quasi-steady manner. The nonlinear aerodynamic damping effect is also included in the analysis. The time-varying standard deviation (STD) and kurtosis of response are determined from the statistical moment equations using non-Gaussian closure technique. The effectiveness of the analysis framework is examined through comparison with response time history simulations. The characteristics of nonstationary crosswind response are also discussed.

Introduction

Tall buildings and other flexible structures such as chimney and towers tend to be more flexible and more sensitive to crosswind loading caused by vortex shedding. With a decrease in structural frequency, the crosswind response at the vicinity of vortex lock-in wind speed needs to be carefully studied. The time variation of crosswind response is between a sinusoidal variation and stochastic process and has a lower peak factor than that of traditional buffeting response. This unique response character is related to the hardening non-Gaussian response distribution caused by nonlinear aerodynamic damping effect [2-4]. Chen [2,4] presented analytical solutions of crosswind response statistics under stationary excitation using equivalent nonlinear equation (ENLE) approach. The extensive studies in literature on crosswind loads and their effects on tall buildings have provided better understanding of response characteristics under stationary wind.

This study presents an analytical framework for estimating stochastic crosswind response of tall buildings with nonlinear aerodynamic damping under nonstationary wind excitations. The statistical moment equations of the building motion including time-varying respone STD and kurtosis are solved using non-Gaussian closure technique. The effectiveness of the analysis framework and the nonstationary response characteristics are examined through comparison with response time history simulation.

Analytical framework

Equation of crosswind response

The crosswind response of a tall building under nonstationary wind excitation is represented in fundamental modal response. The nonstationary wind field is characterized by a time-varying mean wind speed with a time-invariant velocity profile and turbulence intensity profile. It is assumed that the variation of mean wind speed is not rapid such that the aerodynamic buffeting and self-excited forces can be modelled using the force characteristics under stationary wind but with consideration of time-varying mean wind speed in a quasi-steady manner. The equation of motion in terms of non-dimensional displacement is represented as [2]

$$\ddot{y} + 2\omega_s(\xi_s + \xi_a)\dot{y} + \omega_s^2 y = Q(t) \tag{1}$$

$$Q(t) = \frac{1}{2}\left(\frac{\rho B^2}{m_s}\right)\left(\frac{U^2}{B^2}\right)\eta_b \eta C_{Mb}(t) \tag{2}$$

$$\xi_a = -\frac{1}{4}\left(\frac{\rho B^2}{m_s}\right)\eta_{se}\eta H_1^* \tag{3}$$

$$m_s = \frac{\int_0^H m(z)\phi^2(z)dz}{\int_0^H \phi^2(z)dz}; \quad \eta = \frac{H}{\int_0^H \phi^2(z)dz} \tag{4}$$

where $\omega_s = 2\pi f_s$ and ξ_s are generalized modal frequency and damping ratio; m_s is effective building mass per unit height; $m(z)$ is building mass per unit height; $y = y_1/B$; y_1 is generalized displacement, and is the building top displacement when mode shape $\phi(z)$ is normalized as $\phi(H) = 1$; ξ_a is aerodynamic damping ratio; η is a non-dimensional parameter related to mode shape, and $\eta = 3$ in the case of linear mode shape; η_{se} and η_b are mode shape correction factors. For the linear mode shape, i.e., $\phi(z) = z/H$, $\eta_{se} = \eta_b = 1$; ρ is air density; U is wind speed at building top; B is building width; H is building height; and $C_{Mb}(t)$ is buffeting component of base bending moment coefficient, which is determined by high-frequency-force-balance (HFFB) measurement in wind tunnel.

The aerodynamic damping ratio ξ_a can be determined from forced-vibration test in wind tunnel with harmonic motion. At a given reduced frequency $K = fB/U$, ξ_a is a nonlinear function of vibration amplitude $A = y_{max}$, and can be expressed as follows for $\xi_{a1} = m_s\xi_a/\rho B^2$ [2]:

$$\xi_{a1}(K, y_{max}) = a_1(K) + a_2(K)y_{max} + a_3(K)y_{max}^2 \tag{5}$$

where a_1, a_2 and a_3 are coefficients. In the case of nonstationary wind excitation, these coefficients are time dependent due to time-varying mean wind speed.

For stochastic response analysis, the aerodynamic damping needs to be expressed as a nonlinear function of time-varying velocity and/or displacement as [2]

$$\xi_{a1}(K, \dot{y}) = A_1(K) + \frac{A_2(K)}{K}\left(\frac{B|\dot{y}|}{U}\right) + \frac{A_3(K)}{K^2}\left(\frac{B\dot{y}}{U}\right)^2 \tag{6}$$

where $A_1 = a_1$, $A_2 = 3\pi a_2/8$, and $A_3 = 4a_3/3$.

The equation of motion is represented in a state-space format:

$$\dot{\boldsymbol{q}} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}Q(t) \tag{7}$$

$$\boldsymbol{q} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}; \quad \boldsymbol{D} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad \boldsymbol{g}(\boldsymbol{q}) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -2\omega_s\xi_s\dot{y} - \frac{2\omega_s\rho B^2}{m_s}\left(A_1 + \frac{A_2}{K}\left(\frac{B|\dot{y}|}{U}\right) + \frac{A_3}{K^2}\left(\frac{B\dot{y}}{U}\right)^2\right)\dot{y} - \omega_s^2 y \end{bmatrix} \tag{8}$$

Crosswind response under self-excited force only

The crosswind response without consideration of the effect of buffeting force can be determined from the time domain solution of Eq. (7). Alternatively, Eq. (1) leads to the following equation for the vibration amplitude $A(t) = y_{max}(t)$:

$$\dot{A}(t) = -(\xi_s + \xi_a(A))\omega_s A(t) \tag{9}$$

Under the time-varying mean wind speed, the vibration amplitude $A(t)$ is lower than the steady-state vibration amplitude, i.e., amplitude under constant wind speed $U = U(t)$, which is determined by setting the total system damping as zero, i.e. $\xi_s + \xi_a(A) = 0$.

Statistics of stochastic crosswind response

The stochastic crosswind response can be modelled as the response under white-noise load excitation. According to state-space equation, Eq. (7), the covariance equation can be written as (e.g. [6]):

$$\dot{\boldsymbol{R}}_{\boldsymbol{qq}} = \boldsymbol{R}_{\boldsymbol{gq}} + \boldsymbol{R}_{\boldsymbol{gq}}^T + 2\pi \boldsymbol{D}\boldsymbol{D}^T S_0(t) \tag{10}$$

where $\boldsymbol{R}_{\boldsymbol{qq}} = E[\boldsymbol{q}\boldsymbol{q}^T]$ and $\boldsymbol{R}_{\boldsymbol{gq}} = E[\boldsymbol{g}\boldsymbol{q}^T]$ are the covariance matrixes; T is transpose; and $S_0(t)$ is given in one-sided power spectrum of $C_{Mb}(t)$ at the structural frequency, $S_{CMb}(f_s, t)$, as

$$S_0(t) = \frac{1}{16\pi}\left(\frac{\rho B^2}{m_s}\right)^2 \left(\frac{U(t)^2}{B^2}\right)^2 \eta_b^2 \eta^2 S_{CMb}(f_s, t) \tag{11}$$

The quasi-stationary response STD is determined by setting $\dot{\boldsymbol{R}}_{\boldsymbol{qq}} = \boldsymbol{0}$. For a linear system with a constant damping, the quasi-stationary STD of displacement can be calculated as:

$$\sigma_y^2(t) = \frac{\pi S_0(t)}{2\xi_s \omega_s^3} \tag{12}$$

In the case of system with nonlinear aerodynamic damping, the quasi-stationary response STD can also be calculated by ENLE approach [2].

The solution of Eq. (10) involves the statistical moments higher than second order. Similarly, the higher moments equations can also be presented that involve even higher moments. To solve the moment equations, these higher moments need to be represented in terms of unknown lower moments. This technique is referred as closure technique (e.g., [7]). When the higher moments are determined by assuming $\boldsymbol{q}$ follows joint Gaussian distribution, this approach is called Gaussian closure, which is also identical to statistical linearization approach. As the crosswind response with nonlinear aerodynamic damping has a non-Gaussian distribution, a non-Gaussian closure technique should be used for a better estimation. The response is considered as unskewed hardening non-Gaussian process. The higher moments can be estimated by Hermite translation model with given response kurtosis [5]. The additional equations for $E[y^4]$ and $E[\dot{y}^4]$ are determined by Markov process theory (e.g., [7]):

$$\frac{dE[y^4]}{dt} = 4E[y^3 g_1] \tag{13}$$

$$\frac{dE[\dot{y}^4]}{dt} = 4E[\dot{y}^3 g_2] + 12\pi S_0 E[\dot{y}^2] \tag{14}$$

Combining Eqs. (10), (13)-(14), the time-varying statistical moments $E[y^2], E[y\dot{y}], E[\dot{y}^2], E[y^4]$ and $E[\dot{y}^4]$ can be solved.

In the case of stationary excitation, $E[y\dot{y}] = E[y^3\dot{y}] = 0$. Also $\sigma_{\dot{y}}^2 = \omega_s^2 \sigma_y^2$, $E[\dot{y}^4] = \omega_s^4 E[y^4]$ and $E[y\dot{y}^3] = 0$ by using narrowband feature. Eqs. (10), (13)-(14) can be simplified for unknown $E[y^2]$ and $E[y^4]$:

$$(1 + A_1^*)E[y^2] + A_2^* E[y^2|y|] + A_3^* E[y^4] - \frac{\pi S_0}{2\xi_s \omega_s^3} = 0 \tag{15}$$

$$(1 + A_1^*)E[y^4] + A_2^* E[y^4|y|] + A_3^* E[y^6] - \frac{3\pi S_0 E[y^2]}{2\xi_s \omega_s^3} = 0 \tag{16}$$

where $A_1^* = A_1/S_{cr}$, $A_2^* = A_2/S_{cr}$, $A_3^* = A_3/S_{cr}$ and $S_{cr} = m_s \xi_s / \rho B^2$. When Gaussian closure is used, Eq. (16) satisfies automatically. It is note that in the case of linear system, i.e., $A_1 = A_2 = A_3 = 0$, Eqs. (15) and (16) result in the well-known formula for steady-state σ_y^2 with kurtosis of 3.

Results and discussions

Crosswind response without aerodynamic damping effect

In this study, a square-shaped tall building in a very smooth terrain is considered as an example. The aspect ratio of building H/B=13.3. The mass parameter $m_s/\rho B^2 = 172$ and building damping ratio $\xi_s = 1\%$. The fundamental modal shape is assumed as linear over the building height. The aerodynamic damping ratio as function of vibration amplitude at different reduced wind speed is calculated using the model introduced by Watanabe et al. [9] that was developed based on wind tunnel data [8]. The STD value and power spectrum of the based bending moment coefficient are determined using the model recommended by Archetectual Institute of Japan [1]. The time-varying mean wind speed is modelled as

$$U(t) = U_{max} \bar{d}(t) \tag{17}$$

$$\bar{d}(t) = \exp[-(t - t_0)^2 / 2D_t^2] \tag{18}$$

where U_{max} is maximum mean wind speed at the building top; $\bar{d}(t)$ is the time-varying modulation functions for the mean wind speed; t_0 is the time instant at which $\bar{d}(t)$ reaches its maximum value; and D_t is wind storm duration parameter. A smaller value of D_t indicates a short duration of storm, i.e., a larger variation of mean wind speed.

To validate the analytical approach, response history simulation is also performed through the solution of state-space equation using Runge-Kutta method. The time step is 0.04 s and duration is 10 min for each sample. The time history of $C_{Mb}(t)$ is generated by using spectral representation method for nonstationary process. The time-varying statistics are calculated by ensemble average of 1000 simulated samples.

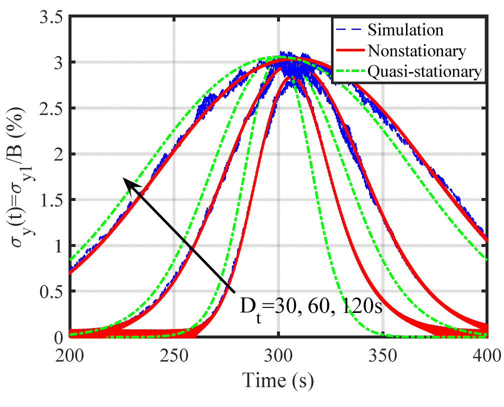


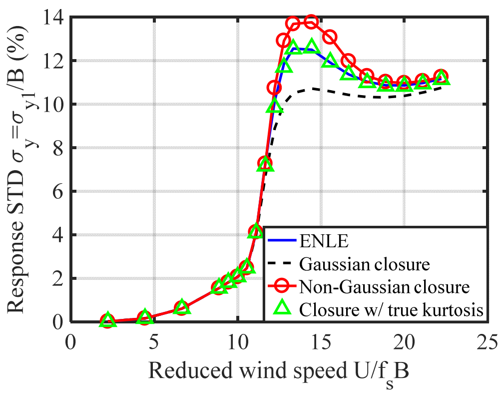
Fig. 1 Crosswind response without aerodynamic damping ($U_{max}/U_{cr} = 0.9$)

Fig. 1 shows the crosswind response without aerodynamic damping under nonstationary excitation with $U_{max}/U_{cr} = 0.9$, where U_{cr} is critical wind speed for vortex induced vibration and $U_{cr}/f_s B = 11.1$. The time-varying STD of linear system can be accurately estimated by solving the covariance equation directly.

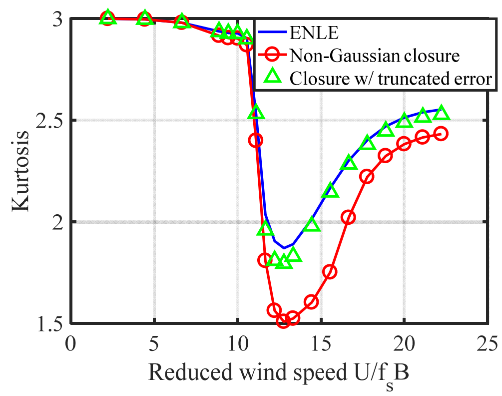
The response STD under nonstationary excitation is lower than that of quasi-stationary response due to transient effect, especially when D_t is shorter. The transient effect also results in a time lag such that the response STD as a function of wind speed is not symmetrical about $t = 300$ s as the wind speed does. The maximum response and peak factor under nonstationary wind excitation are much lower than those of stationary wind due to the fact that the large response is only developled for a shorter time duration.

Crosswind response with nonlinear aerodynamic damping under stationary wind excitation

Chen [2] showed that the crosswind response with aerodynamic damping effect under stationary wind excitation can be accurately estimated by using ENLE approach. Fig. 2 compares the response statistics under stationary wind excitation by using ENLE approach, Gaussian and non-Gaussian closure techniques. The result from Gaussian closure is identical to that from statistical linearization with Gaussian assumption (e.g. [7]). The results show the STD estimated by using Gaussian closure is lower compared with that from ENLE approach. The non-Gaussian character introduced by nonlinear aerodynamic damping can be captured, to some extent, by using non-Gaussian closure technique, but the estimated response STD and kurtosis are slightly higher and lower than those from ENLE approach, respectively.



a) Response STD



b) Kurtosis

Fig. 2 Crosswind response under stationary wind excitation

When the true kurtosis calculated from ENLE approach is used to estimate higher moments in Eq. (15) through Hermite translation model, the response STD can be accurately estimated. It demonstrates the accuracy of Eq. (15) in estimating response STD. On the other hand, when the moments in Eq. (16) are calculated from the ENLE approach and it is found that the equation is only approximately valid. When this truncation error is further accounted and the kurtosis is recalculated from Eq. (16) where the higher momoents are determined from kurtosis-based Hermite translation model, the estimation of kurtosis can be improved. The estimation error of non-Gauissian closure technique is attributed to the approximation of Eq. (16) and the error of modelling higher moments from kurtosis with Hermite translation model. Further development of quantificantion of kurtosis is needed.

Crosswind response with nonlinear aerodynamic damping under nonstationary wind excitation

For the case of nonstationary excitation, crosswind response under self-excited force only is examined firstly. Fig. 3 shows the time history of displacement with the initial condition of $y = 2\%$ and $\dot{y} = 0$ at t=250 s. The vibration amplitude agrees with that directly determined from Eq. (9), and is much lower than the quasi-stationary amplitude.

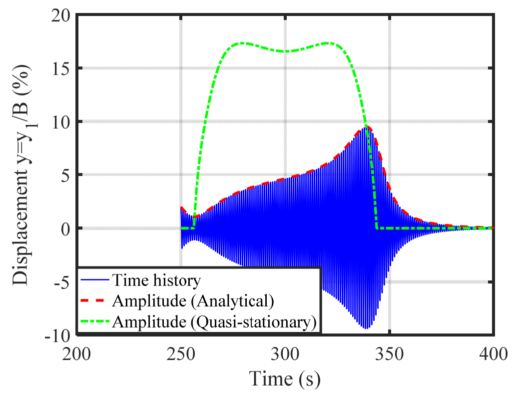


Fig. 3 Displacement of self-excited vibration under nonstationary excitation ($U_{max}/f_s B = 14.4,\ D_t = 60$s)

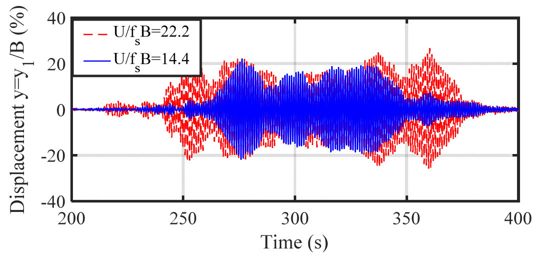
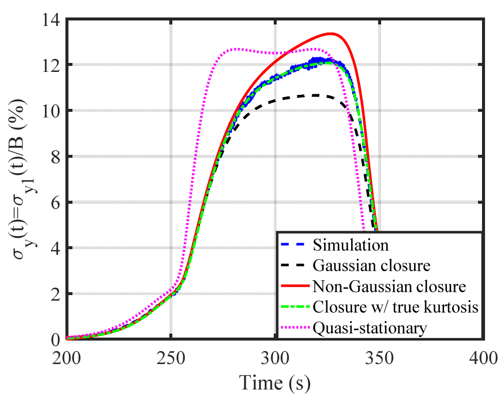
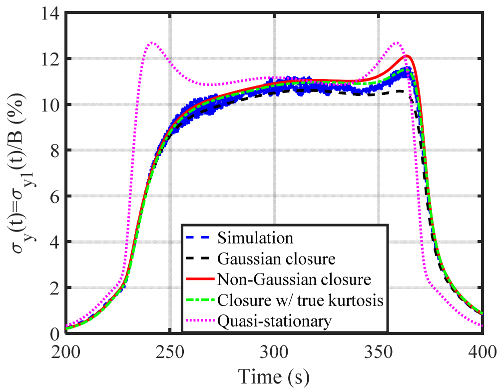


Fig. 4 Time history sample of crosswind displacement under nonstationary wind excitation ($D_t = 60$s)

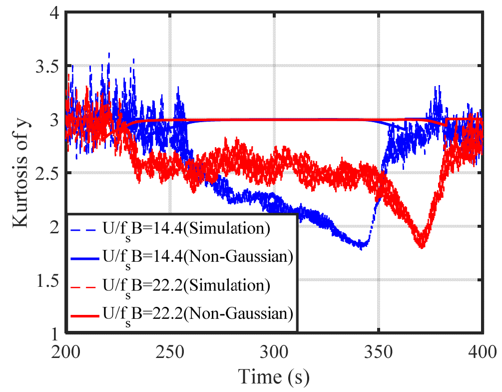


a) $U_{max}/f_s B = 14.4$

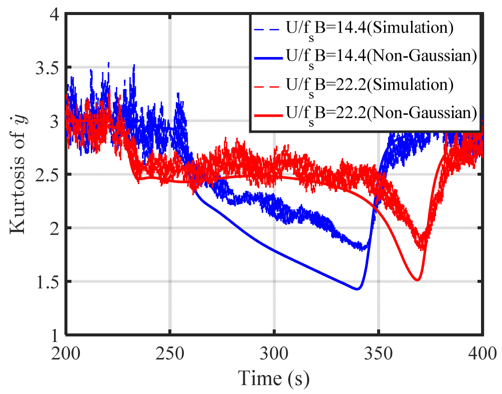


b) $U_{max}/f_sB = 22.2$

Fig. 5 Time-varying STD of crosswind displacement ($D_t = 60$s)



a) Kurtosis of displacement



b) Kurtosis of velocity

Fig. 6 Time-varying kurtosis ($D_t = 60$s)

In the case of stochastic response under nonstationary wind excitation, time history samples with $D_t = 60$s at $U_{max}/f_sB = 14.4$ and 22.2 are shown in Fig. 4. The corresponding time-varying STD and kurtosis of displacement are shown in Figs. 5 and 6, which are determined from 1000 samples. The results with non-Gaussian and Gaussian closure techniques are also given. It is observed that the time-varying response STD can be accurately calculated from covariance equation, i.e. Eq. (10), when the time-varying kurtosis of y and $\dot{y}$ from time history analysis are used with the non-Gaussian closure technique. As expected, the Gaussian closure approach is less accurate as the response shows clearly hardening non-Gaussian character. The response STD is also not well estimated in non-Gaussian closure approach because of the error in the estimated response kurtosis. Compared to the case of $U_{max}/f_sB = 14.4$, in the case of $U_{max}/f_sB = 22.2$, the non-Gaussian closure technique gives a better estimation of response STD due to the fact that non-Gaussian feature is less significant when wind speed is far away from vortex lock-in wind speed as shown in Fig. 2a). The non-Gaussian character of displacement cannot be captured by the non-Gaussian closure technique as shown in Fig. 6a). This might due to the ineffectiveness of Eqs. (13) and (14), and this issue is now under further investigation. It is also noted that the response STD is less than the quasi-stationary estimation due to transient effect. The cases of other time duration D_t give the similar result, thus are not further discussed here.

Conclusions

The analytical approach for estimating crosswind response of tall buildings at the vicinity of vortex lock-in speed under nonstationary wind excitation was examined. The non-Gaussian closure approach can lead to accurate estimation of time-varying STD of response when true response kurtosis is used. However, the challenge remains on the accurate estimation of response kurtosis when response shows stronger non-Gaussian character, which then affects the estimated response STD. The response under nonstationary wind excitation is lower than that of stationary case due to transient dynamic effect.

Acknowledgments

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