A Tale of Two Delays: Disentangling Loop Behavior In the North Atlantic Current

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1. AMOC

The Atlantic Meridional Overturning Circulation (AMOC) is a large ocean current system that carries warm/salty water from the tropics northwards into the North Atlantic. Changes in the dynamics of AMOC can have, as a consequence, significant changes in the global climate.

Different types of mechanisms can explain the AMOC dynamics. We focus on a large-scale Thermocline Circulation model (THC) on a long time scale, concentrating on water transport due to density disparities caused by differences in salinity and control by two independent feedback loops, providing the ocean currents with the energy necessary to sustain a stable deep overturning circulation.

$$\frac{dS(t)}{dt} = \frac{\Psi}{S_0} S(t-\tau)(S_e - S(t-\sigma)) \qquad (1)$$

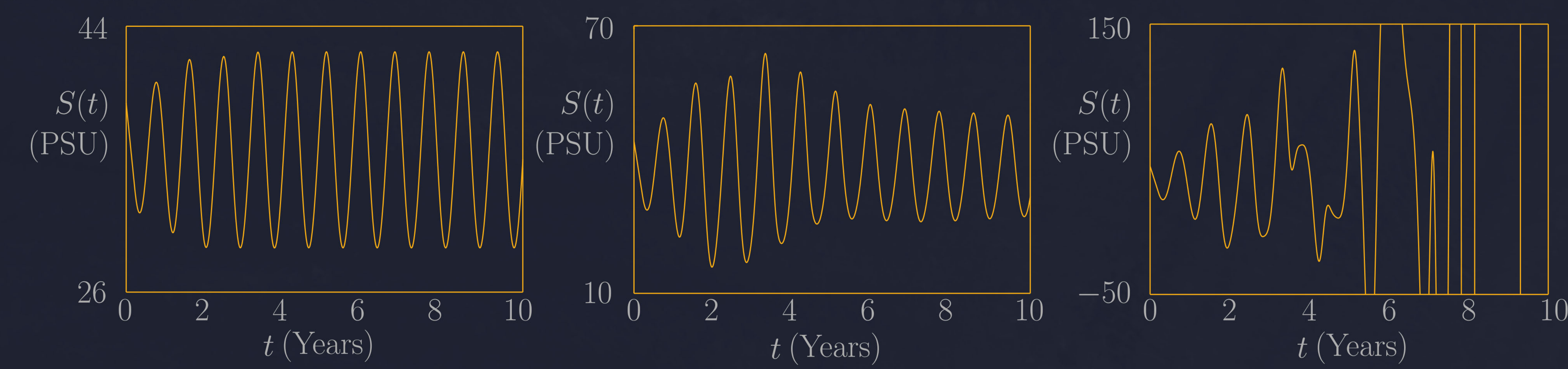
$$x'(t) = x(t-\tau)(1-x(t-\sigma))$$

2. The model

Modeling AMOC considering the feedback loops explicitly as delay times rest on the idea of reducing the vast amount of processes on the system to a single phenomena base on the delayed effect. Moreover, considering the explicit inclusion of the delayed effects can lead to a more precise description

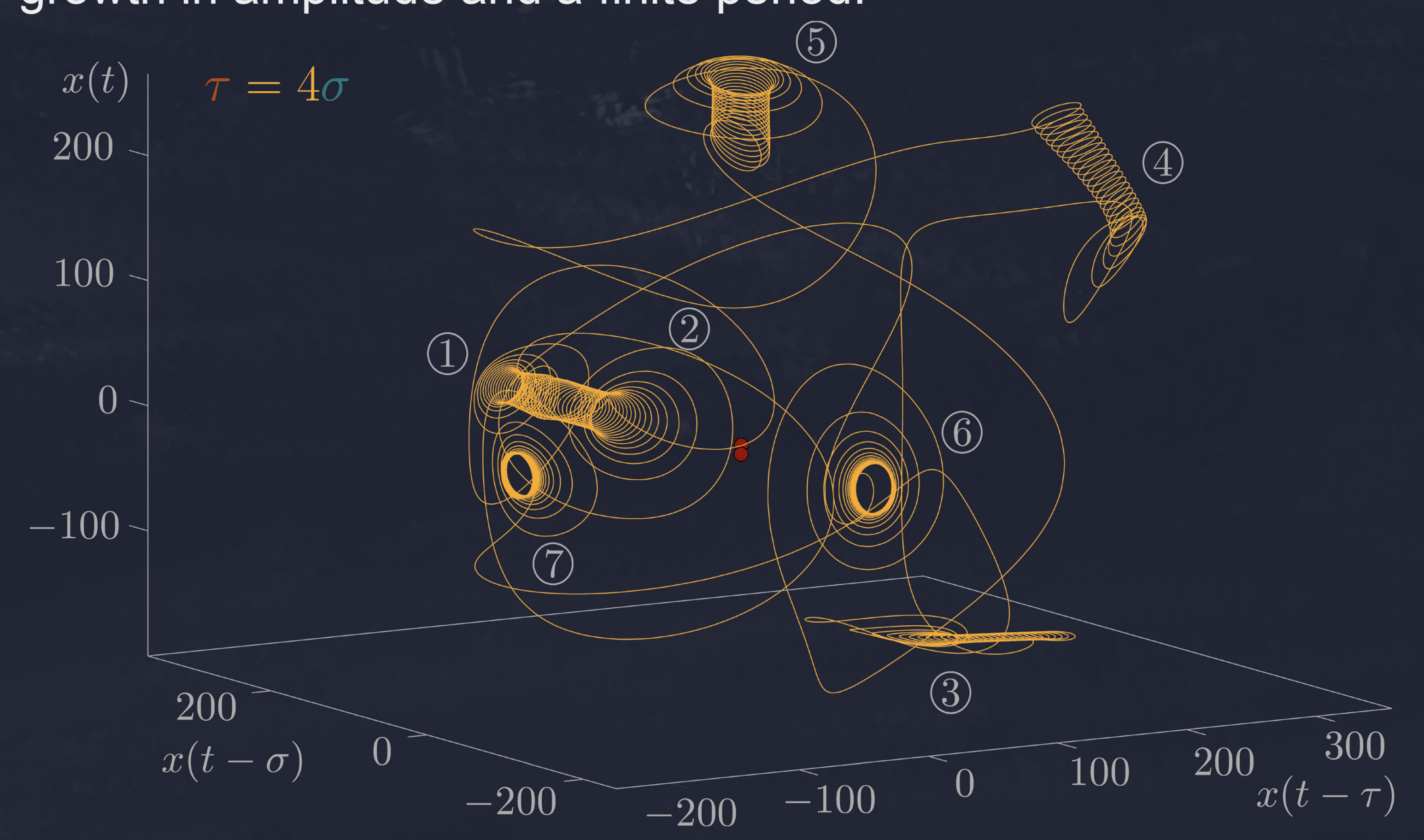
3. Stabilty of solutions

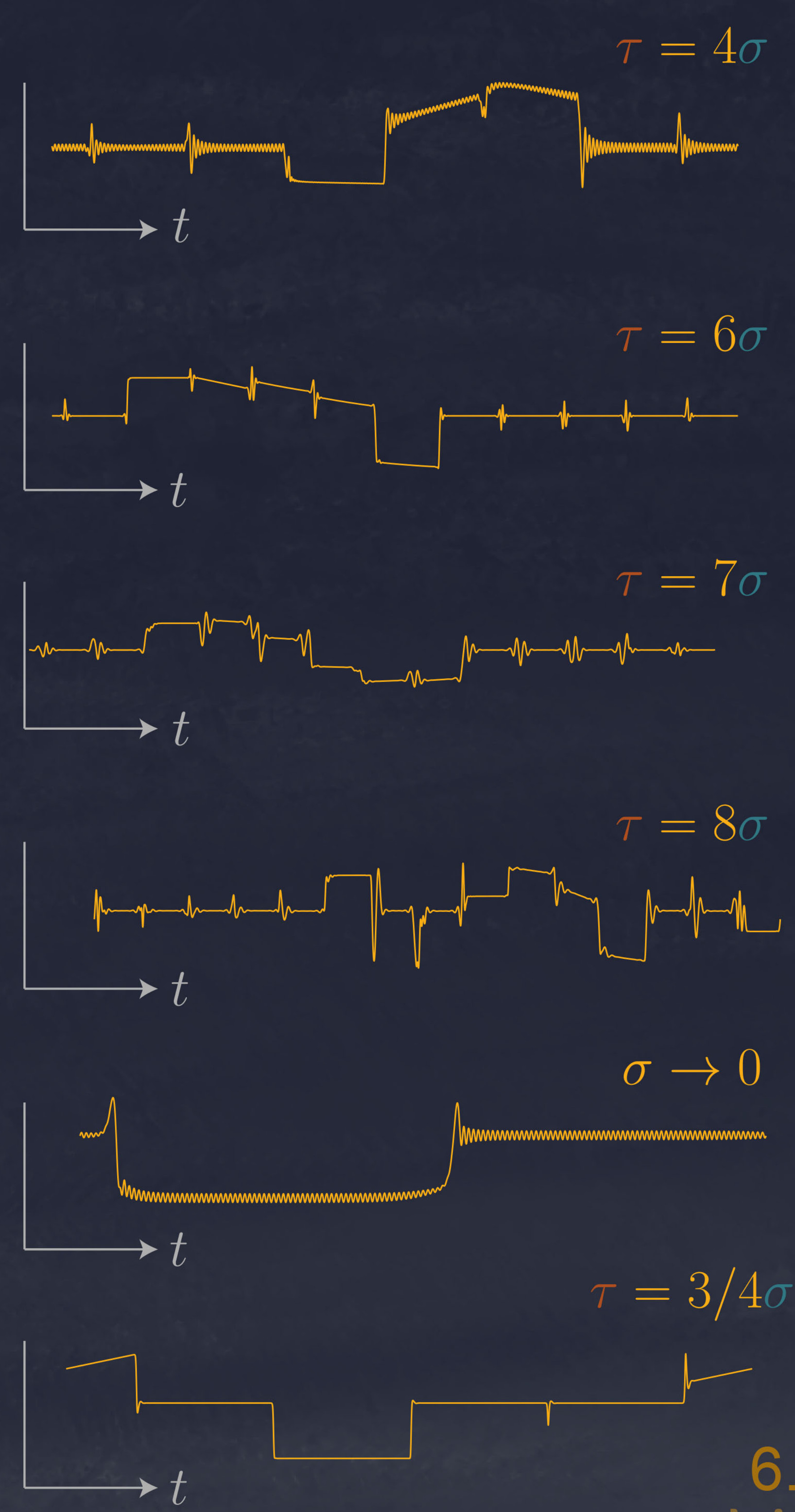
The main focus of our research is an exploration and in-depth analysis of the dynamics exhibited by Eq (1) for different choices of the two remaining independent parameters τ and σ, and different initial conditions.



4. Resonance phenomena between the delays

A novel phenomena caused by the interaction of the delays were observed when the delays have a proportional value $\tau = k\sigma$. These phenomena are characterized by complicated saddle periodic solutions with unique characteristics, such as an unbound growth in amplitude and a finite period.





This resonance phenomena give rise to many questions, such as the region of existence, the cause of the unlimited growth in amplitude, the shape of the solution as we approach the resonance proportions, among others.

5. Resonance Distribution

In order to investigate the distribution of these resonant solutions, we can make use of two properties of (1).

1) Reappearence

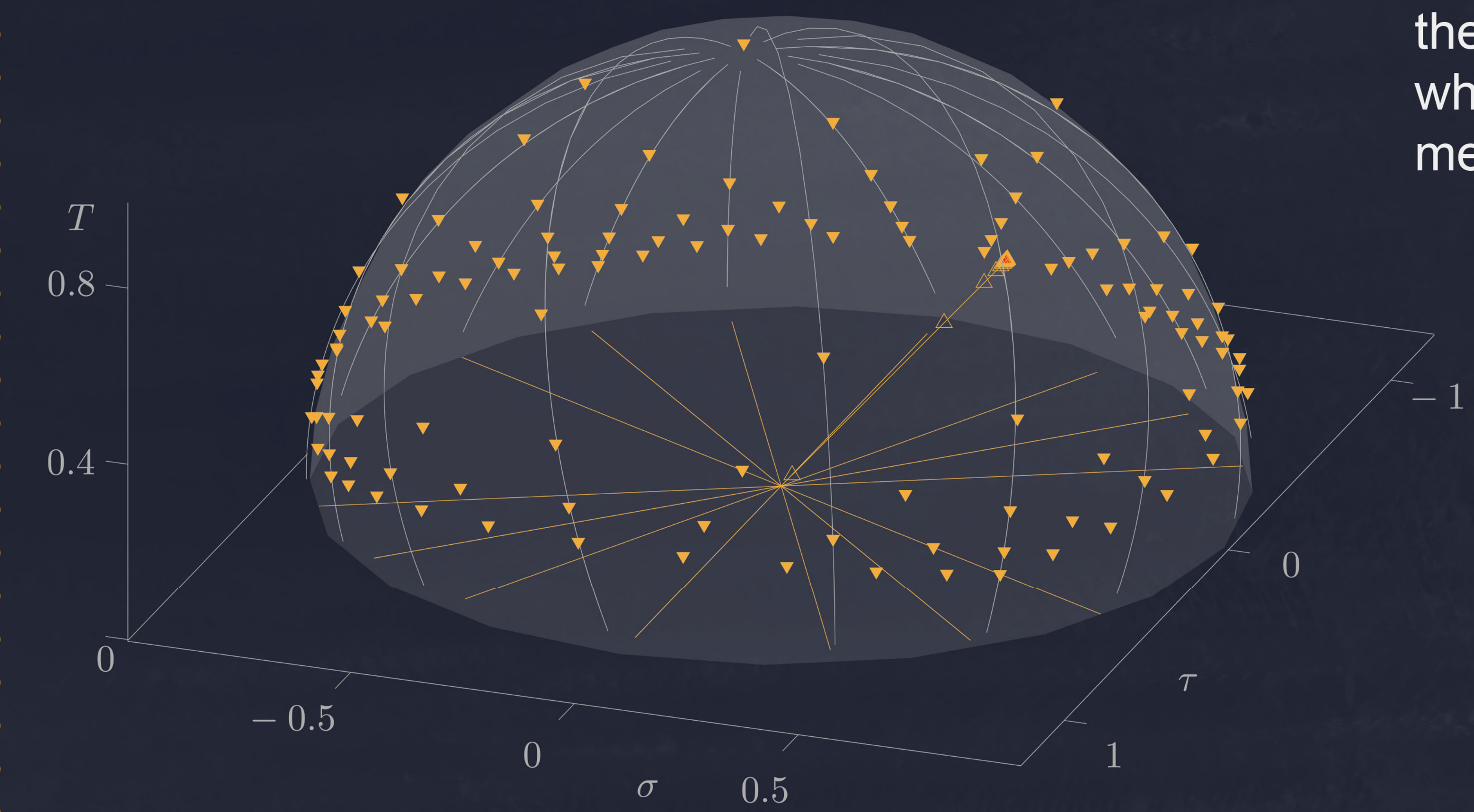
$$x'(t) = f[x(t), x(t-\tau)] = f[x(t), x(t-\tau-nT)]$$

It means that we obtain the same periodic solution but for proportional values of the delays.

2) Symmetry

$$(x, t, \tau, \sigma) \longmapsto (1-x, -t, -\sigma, -\tau).$$

This implies that for every solution of (1) at parameters values (τ, σ) there exist a solution at $(-\sigma, -\tau)$, which is a conjugate under the symmetry action.



The distribution of the locking $\tau = 4\sigma$ and its corresponding reappearance after and before applying the symmetry indicate the existence of the periodic solution in all the space.

6. Future Work

One of the main remaining challenges is extending and comprehending the dynamics across the complete physical space, which poses a significant challenge, requiring a thorough analysis of the system's stability. Additionally, a crucial factor to incorporate into the system involves the impact of seasonal variations on the AMOC. These variations, stemming from seasonal changes, introduce an external forcing that influences the AMOC's behavior.

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