1 # MATHS 208 Exam 2020S1 Q2V3

Suppose that f is a function of two variables and that $\nabla f = (1, -y)$. Suppose (a, b) is some point. Which of the following vectors $\mathbf{u}$ is a unit vector such that $D_{\mathbf{u}} f(a, b) = 0$?

Select one alternative:

- $\mathbf{u} = (b, -1)$
- $\mathbf{u} = \left(\dfrac{b}{\sqrt{1+b^2}}, \dfrac{1}{\sqrt{1+b^2}} \right)$
- $\mathbf{u} = \left(\dfrac{1}{\sqrt{a^2+1}}, \dfrac{a}{\sqrt{a^2+1}} \right)$
- $\mathbf{u} = \left(\dfrac{1}{\sqrt{a^2+b^2}}, \dfrac{-a}{\sqrt{a^2+b^2}} \right)$

Maximum marks: 1

2 MATHS 208 Exam 2021S1 Q1V3

Suppose that z is implicitly defined as a function of x and y by the equation $xz + xy + \sin(x + 3y) = 3x + y$. Consider the two proposed attempts at computing $\frac{\partial z}{\partial x}$, and decide whether they are correct.

Attempt 1:

We let $F(x, y, z) = xz - xy + \sin(x + 3y)$, so
$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{z - y + \cos(x + 3y)}{x}.$$

Attempt 2:

Differentiate with respect to x, treating y as a constant, to get
$x\frac{\partial z}{\partial x} - y + \cos(x + 3y) = 3$, and rearrange to get
$$\frac{\partial z}{\partial x} = \frac{-y - \cos(x + 3y) + 3}{x}.$$

Select one alternative

- Attempt 1 is correct and Attempt 2 is correct.
- Attempt 1 is incorrect and Attempt 2 is correct.
- Attempt 1 is correct and Attempt 2 is incorrect.
- Attempt 1 is incorrect and Attempt 2 is incorrect.

Maximum marks: 1

3

MATHS 208 Exam 2021S1 Q3V3

Suppose that a is a real number and that f is a function of two variables given by $f(x, y) = e^{-x} - xy + ay$.

Which of the following correctly describes a critical point of f?

Select one alternative:

- f has a local maximum at the point $(x, y) = (a, e^{a})$
- f has a saddle point at the point $(x, y) = (a, -e^{-a})$
- f has a local maximum at the point $(x, y) = (a, -e^{-a})$
- f has a saddle point at the point $(x, y) = (a, e^{a})$

Maximum marks: 1

4 MATHS 208 Exam 2020S1 Q4V2

At which one of the following points does the function $f(x, y) = x + 5y - 1$ achieve a relative maximum value subject to the constraint $e^x + e^y = 1$?

Select one alternative

- $(x, y) = (-\ln(5), \ln(4) - \ln(5))$
- $(x, y) = (-\ln(6), \ln(5) - \ln(6))$
- $(x, y) = (0, \ln(5))$
- $(x, y) = (-\ln(6), \ln(6) - \ln(5))$

Maximum marks: 1

5 MATHS 208 Exam 2020S1 Q5V3

Suppose that g is a function of three variables. Suppose that the function given by $f(x, y, z) = x + z$ attains a maximum value and a minimum value subject to the constraint $g(x, y, z) = 0$.

Suppose that at the point $(x, y, z) = (2, 5, 0)$,$\nabla g = (-1, 0, -1)$.
Suppose that at the point $(x, y, z) = (7, 2, 3)$,$\nabla g = (4, 4, 4)$.

Based on this information, decide which of the following two statements are correct.

Statement 1:
The maximum value of $x + z$ subject to $g(x, y, z) = 0$ <u>must</u> occur at the point $(x, y, z) = (2, 5, 0)$.

Statement 2:
The maximum value of $x + z$ subject to $g(x, y, z) = 0$ <u>cannot</u> occur at the point $(x, y, z) = (7, 2, 3)$.

Select one alternative:

- Statement 1 is correct and statement 2 is correct.
- Statement 1 is incorrect and statement 2 is incorrect.
- Statement 1 is incorrect and statement 2 is correct.
- Statement 1 is correct and statement 2 is incorrect.

Maximum marks: 1

6 # MATHS 208 Exam 2020S1 Q6V3

Consider the two methods for checking whether the sequence $\{a_n\}_{n=1}^{\infty}$ given by $a_n = \dfrac{6n - \cos(n)}{2n}$ converges, and decide whether they are carried out correctly.

Method 1:

We apply the Squeezing Theorem:
$\dfrac{6n-1}{2n} \leq \dfrac{6n-\cos(n)}{2n} \leq \dfrac{6n+1}{2n}$ for each n, and
$\lim\limits_{n\to\infty} \dfrac{6n-1}{2n} = \lim\limits_{n\to\infty} \dfrac{6n+1}{2n} = 3$. Therefore
$\lim\limits_{n\to\infty} \dfrac{6n-\cos(n)}{2n} = 3$ as well, so the sequence converges.

Method 2:

We apply l'Hopital's Rule:
$\lim\limits_{n\to\infty} \dfrac{6n-\cos(n)}{2n} = \lim\limits_{n\to\infty} \dfrac{6+\sin(n)}{2} = 3$. Therefore the sequence converges.

Select one alternative:

- Method 1 is correct and method 2 is incorrect.
- Method 1 is incorrect and method 2 is incorrect.
- Method 1 is correct and method 2 is correct.
- Method 1 is incorrect and method 2 is correct.

Maximum marks: 1

7 MATHS 208 Exam 2020S1 Q7V2

Consider the two series $\sum_{n=1}^{\infty} \frac{n}{3^n}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$. For each series, decide whether the ratio test tells us the series converges.

Select one alternative:

- The ratio test does not tell us that the series $\sum_{n=1}^{\infty} \frac{n}{3^n}$ converges, but tells us that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$ converges.

- The ratio test tells us that the series $\sum_{n=1}^{\infty} \frac{n}{3^n}$ converges and tells us that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$ converges.

- The ratio test does not tell us that the series $\sum_{n=1}^{\infty} \frac{n}{3^n}$ converges, and it does not tell us that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$ converges.

- The ratio test tells us that the series $\sum_{n=1}^{\infty} \frac{n}{3^n}$ converges, but it does not tell us that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$ converges.

Maximum marks: 1

8

MATHS 208 Exam 2020S1 Q8V2

Suppose that $|r| < 1$. Which of the following is equal to $\displaystyle\sum_{n=2}^{\infty} \frac{r^n}{7}$?

Select one alternative:

- $\dfrac{1}{7-r}$
- $\dfrac{1}{1-7r} - \dfrac{1}{7} - \dfrac{1}{49}$
- $\dfrac{1}{7-7r}$
- $\dfrac{1}{7(1-r)} - \dfrac{1}{7} - \dfrac{r}{7}$

Maximum marks: 1

9 # MATHS 208 Exam 2020S1 Q9V1

Recall that $\sum_{n=0}^{\infty} x^n = \dfrac{1}{1-x}$ for $-1 < x < 1$.

Which of the following power series is equal to $\dfrac{1}{1-2(x-3)}$ for the x-values in the specified interval?

Select one alternative:

- $\sum_{n=0}^{\infty} (2(x-3))^n = \dfrac{1}{1-2(x-3)}$ for $x < \frac{7}{2}$.
- $\sum_{n=0}^{\infty} 2^n (x-3)^n = \dfrac{1}{1-2(x-3)}$ for $\frac{5}{2} < x < \frac{7}{2}$.
- $\sum_{n=0}^{\infty} 2(x-3)^n = \dfrac{1}{1-2(x-3)}$ for $0 < x < 6$.
- $\sum_{n=0}^{\infty} \dfrac{(x-3)^n}{2^n} = \dfrac{1}{1-2(x-3)}$ for $-6 < x < 6$.

Maximum marks: 1

10 MATHS 208 Exam 2021S1 Q11V1

Which one of the following is a vector space?

Select one alternative:

- $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x + y + z = 0 \right\}$

- $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} : x_1, x_2 \in \mathbb{R} \right\}$

- $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 : x_4 \geq 0 \right\}$

- $\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 : x_1 + x_2 = 1 \right\}$

Maximum marks: 1

11

MATHS 208 Exam 2021S1 Q12V1

Let A $=\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ -2 & -2 & -1 & -1 & -3 \\ 3 & 3 & 0 & 0 & 3 \\ -4 & -4 & 0 & -1 & -3 \\ 5 & 5 & 0 & 0 & 5 \\ 6 & 6 & 0 & 0 & 6 \end{bmatrix}$, which row reduces to

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which one of the following is NOT true about Col(A)?

Select one alternative:

- Col(A) is a vector space of dimension 3
- Col(A) is a subspace of $\mathbb{R}^5$
- Col(A) is the span of $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 3 \\ -3 \\ 5 \\ 6 \end{bmatrix} \right\}$
- Col(A) is the span of $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right\}$

Maximum marks: 1

12 MATHS 208 Exam 2021S1 Q13V1

Let A$=\begin{bmatrix} 1 & -1 & -1 \\ 2 & -2 & 1 \end{bmatrix}$, which row reduces to $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Which one of the following vectors is NOT in Null(A)?

Select one alternative:

- $\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$
- $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Maximum marks: 1

13 MATHS 208 Exam 2021S1 Q14V1

Suppose U is a 3-dimensional vector space. Assume the set $B = \{v_1, v_2, v_3\}$ is a basis of U.

Which of the following statements are always true?

Statement A: The 2-element sets of vectors from B, $\{v_1, v_2\}, \{v_1, v_3\}$ and $\{v_2, v_3\}$, are all linearly independent sets.

Statement B: The set $\{v_1, v_2\}$ does not span U.

Select one alternative:

- B only
- A only
- Neither A nor B
- Both A and B

Maximum marks: 1

14 MATHS 208 Exam 2021S1 Q15V3

Let C be a 3×3 symmetric matrix. Assume λ_1, λ_2, λ_3 are three distinct eigenvalues of C. Which of the following statements are always true?

Statement A: If v_1, v_2, v_3 are eigenvectors of C corresponding to the three distinct eigenvalues, then the set $\{v_1, v_2, v_3\}$ forms an orthogonal basis of $\mathbb{R}^3$.

Statement B: If u is an eigenvector of C, corresponding to λ_3, and w is an eigenvector of C corresponding to the same eigenvalue λ_3, then $\lambda_1 u + \lambda_2 w$ is an element of the eigenspace of C, corresponding to λ_3.

Select one alternative:

- Neither A nor B
- A only
- Both A and B
- B only

Maximum marks: 1

15 MATHS 208 Exam 2021S1 Q16V3

Let D$=\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Which one of the following is the eigenspace corresponding to the eigenvalue 2?

Select one alternative:

- $\text{Span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$
- $\text{Span}\left\{ \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix} \right\}$
- $\text{Span}\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$
- $\mathbb{R}^3$

Maximum marks: 1

16 MATHS 208 Exam 2021S1 Q17V1

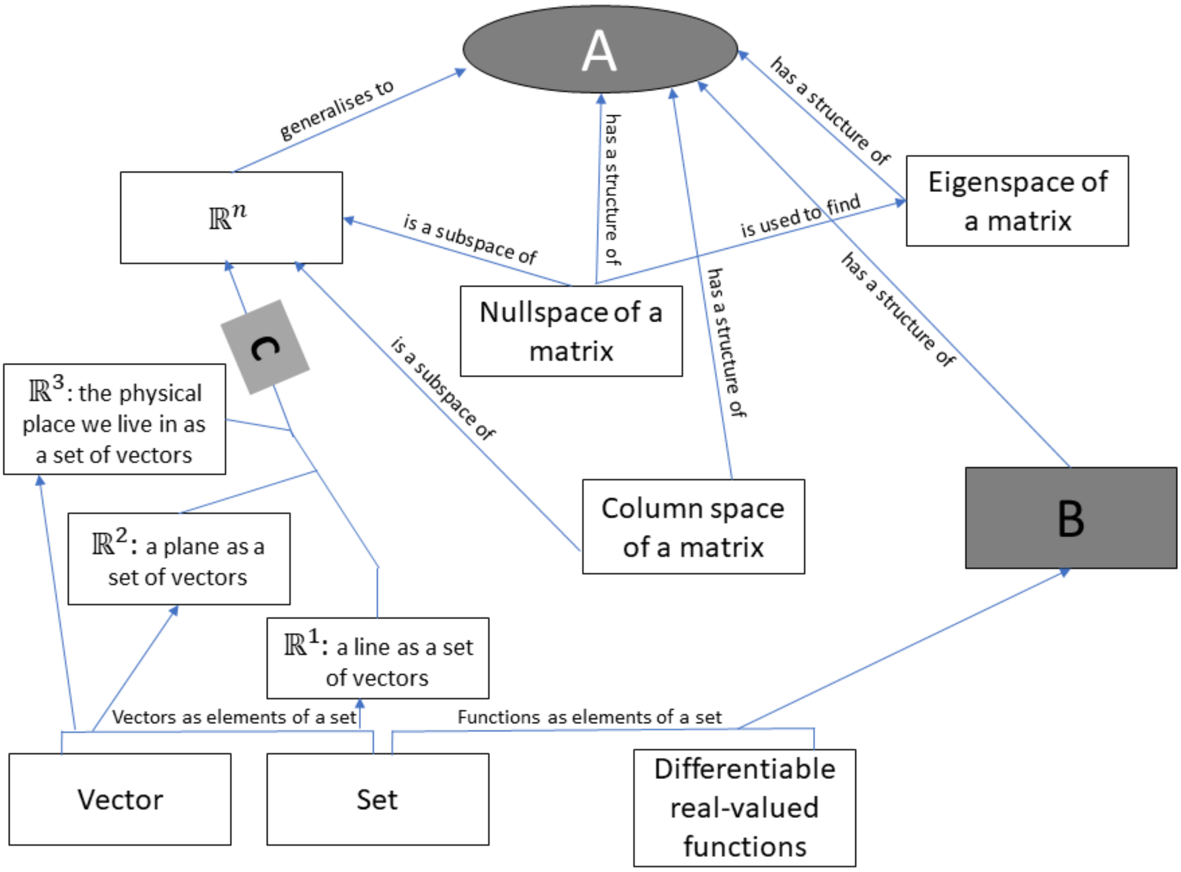
Let A be a 4×4 matrix with eigenvalues $-2, -1, 0, 1.$
Which one of the following is NOT true about eigenvectors of A?
Select one alternative:

- Null(A) contains all eigenvectors of A.
- A has four 1-dimensional eigenspaces.
- Each eigenspace of A is a 1-dimensional subspace of $\mathbb{R}^4$
- If v_1 is an eigenvector of A corresponding to -2 and v_2 is an eigenvector corresponding to 1, then $v_1 + v_2$ is not an eigenvector of A.

Maximum marks: 1

17 MATHS 208 Exam 2021S1 Q18V1

This question has three parts - make sure you answer all of them.



Consider this incomplete concept map. Fill out the three blanks: The main concept **A** in the grey ellipse is ______ (Symmetric matrix, Least squares solutions, Vector space, Discrete dynamical system, Square matrix, Partial derivative, Markov chain, Orthonormal matrix). The concept missing in grey box **B** is ______ (Set of solutions of a linear differential equation, The Wronskian, Normal equations, Series, Least squares solutions, General solution of Ax=b, Sequence). The missing connecting relation labelled by **C** is ______ (generalises to, is a span of, excludes, is a linear combination of)

Maximum marks: 3

18 MATHS 208 Exam 2021S1 Q19V1

Given vectors $\mathbf{u} = \begin{bmatrix} 5 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, which one of the following is the projection of $\mathbf{u}$ onto $\mathbf{v}$?

Select one alternative:

- $\begin{bmatrix} 5 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

- $\begin{bmatrix} -5 \\ -1 \\ -1 \\ -1 \end{bmatrix}$

- $\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

- $\begin{bmatrix} 2 \\ -2 \\ 2 \\ -2 \end{bmatrix}$

Maximum marks: 1

19 # MATHS 208 Exam 2020S1 Q20V3

This question has two parts - make sure you answer both.

On a secret island, two species of dinosaur still exist. One species is a meat-eating species, and the other is a plant-eating species. The meat-eating species hunts the plant-eating species.

Scientists have developed a model which predicts that the populations of the two species change over time according to the equation:

$\begin{bmatrix} p_{n+1} \\ m_{n+1} \end{bmatrix} = \begin{bmatrix} 2.8 & -1.2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} p_n \\ m_n \end{bmatrix}$, where p_n denotes the population of plant-eating dinosaurs and m_n denotes the population of meat-eating dinosaurs (both given n years after the discovery of the island).

The matrix $\begin{bmatrix} 2.8 & -1.2 \\ 3 & -1 \end{bmatrix}$ has eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 0.8$, respectively.

Initially when the island is discovered the populations are $p_0 = 1000$ and $m_0 = 300$.

How many plant-eating dinosaurs will there be two years after the discovery of the island?

Select one alternative:

- 4620
- 2440
- 2700
- 3592

The scientists decided to use their model to predict what will happen to the populations in the long term. Which of the following should describe their conclusion?

Select one alternative

- Both species will become extinct.
- Both species will shrink in number over time. In the long term approximately three-eighths of the dinosaurs present will be plant-eating.
- Neither species will die out, and the populations will both approach a finite nonzero limit over time.
- Both species will shrink in number over time. In the long term approximately two-fifths of the dinosaurs present will be plant-eating.

Maximum marks: 2

20 MATHS 208 Exam 2021S1 Q21V2

Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ be three vectors in $\mathbb{R}^5$ that form a linearly independent set.

Suppose we apply the Gram-Schmidt process to $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ (in this order) to obtain three vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

Decide whether each of the following statements is true.

Statement 1:
$\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

Statement 2:
$\mathbf{v}_3 \cdot \mathbf{u}_1 = 0.$

Select one alternative:

- Statement 1 and Statement 2 are both true
- Statement 1 and Statement 2 are both false
- Statement 1 is false and Statement 2 is true
- Statement 1 is true and Statement 2 is false

Maximum marks: 1

21 MATHS 208 Exam 2021S1 Q22V3

Suppose you are solving a differential equation $\frac{dy}{dx} = 4y - 6x$. You are considering two possible methods.

Method 1:

Rearrange the equation to get it into the form $\frac{dy}{dx} = f(x)g(y)$ and treat it as a separable differential equation.

Method 2:
Identify a suitable integrating factor and treat the DE as a linear differential equation.

Which of these methods will work?
Select one alternative:

- Only method 1 will work.
- Both methods will work.
- Neither method will work.
- Only method 2 will work.

Maximum marks: 1

22 SeparableIVPv1

Solve the initial value problem

$$\frac{dy}{dx} = \frac{5-x}{y}, \quad y(1) = 3$$

for the function $y(x)$. Then calculate the value of $y(8)$, and enter it in the box:

The value of $y(8)$ is ☐.

Maximum marks: 1

23 IntegratingFactorV2

Suppose we want to solve the differential equation $y' + \left(\frac{1}{x} + \frac{1}{x^2}\right) y = 1$ by the method of integrating factors, assuming $x > 0$. Which one of the following is a suitable integrating factor?

Select one alternative:

- $\ln(x)e^{-x}$
- $xe^{-(1/x)}$
- $e^{-x^{-2}-2x^{-3}}$
- $\ln(x) - x^{-1}$

Maximum marks: 1

24 LinhomogDE_V1

Which of the following are linear homogeneous differential equations?

A. $\dfrac{dx}{dy} + \dfrac{x-1}{y} = 0$

B. $\dfrac{d^2w}{dt^2} = (t^2+1)\dfrac{dw}{dt}$

C. $(y+t)\frac{dy}{dt} = 0$

D. $t^2\frac{d^2x}{dt^2} + t\frac{dx}{dt} + x = 0$

Select one alternative:

- $BandD$
- $Donly$
- $A, C, andD$
- $Noneofthem$

Maximum marks: 1

25 2ndOrderLinHomogV1

Which of these expressions is the general solution of some second-order linear homogeneous differential equation $ay'' + by' + cy = 0$ with independent variable x?

A. $y = c_1 x e^x + c_2 e^x$

B. $y = e^{c_1 x} + e^{c_2 x}$

C. $y = c_1 e^{c_2 x}$

D. $y = c_1 e^x + c_2 e^{-x}$

(Here c_1, c_2 are any real numbers.)

Select one alternative:

- D only
- C only
- A and B only
- A and D only

Maximum marks: 1

26 WronskianV2

For which of the following values of a and b is $\{3e^{at}, e^{bt}, e^{t}\}$ a linearly independent set of functions?

Select one alternative:

- $a = 2, b = 1$
- $a = 3, b = 2$
- $a = 1, b = 4$
- $a = 3, b = 3$

Maximum marks: 1

27 Equivalence2ndOrderV2

Consider an initial value problem (IVP) given by a linear homogeneous second order DE and an initial condition $y(0) = a, y'(0) = b$. This IVP can be written as an equivalent system $\mathbf{x}' = A\mathbf{x}$, where $\mathbf{x}(t) = \begin{bmatrix} y(t) \\ y'(t) \end{bmatrix}$, together with the initial condition $\mathbf{x}(0) = \begin{bmatrix} a \\ b \end{bmatrix}$.

Suppose that the solution to the original IVP is $y(t) = e^t - e^{-t}$.
The solution to the equivalent IVP is of the form $\mathbf{x}(t) = e^t\mathbf{v}_1 + e^{-t}\mathbf{v}_2$, for some vectors $\mathbf{v}_1, \mathbf{v}_2$ in $\mathbb{R}^2$.

Which of the following is $\mathbf{v}_1$?

Select one alternative:

- $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

- $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

- $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Maximum marks: 1